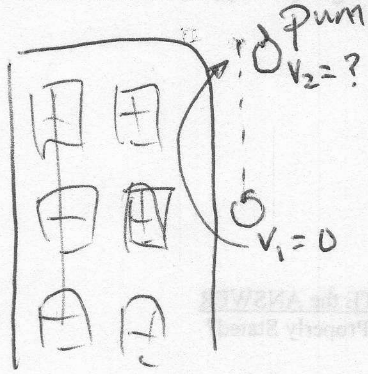
Problem Solving Guidelines

FOCUS the PROBLEM
Picture and Given Information

pumpkin 1 ⇒ dropped pumpkin



Question(s)

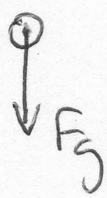
FIND ~~velocity~~ time delay to launch second pumpkin so both hit ground at same time – variable velocity

Approach

Kinematics –

DESCRIBE the PHYSICS
Schematic Diagram(s) (Free Body Diagrams) and Define Variable Quantities

Ignore Air Drag



$a = g$

pumpkin 1 $V_i = 0$

pumpkin 2 $V_0 = -V$

$x_0 = h$

$x_f = 0$

Target Quantity(ies)

$\Delta x = x_f - x_0 = -h$

Quantitative Relationships

$a = g$

$v(t) = \int a\,dt = v_0 + at \quad \Rightarrow \quad v = v_0 - gt$

$x(t) = \int v(t)\,dt = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad 0 = -\Delta x + v_0 t - \frac{1}{2}gt^2$

$0 = h + v_0 t - \frac{1}{2}gt^2$

solve for t

If $v_0 = 0$ $\quad t = \left(\frac{2h}{g}\right)^{1/2}$ $\quad$ If $v_0 \neq 0$ $\quad t = \frac{v_0}{g} \mp \left[\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}\right]^{1/2}$

PLAN the SOLUTION

eq num | unknown

Pumpkin 1

$$t_1 = \left(\frac{2\cancel{\Delta x}}{g}\right)^{1/2} \quad \text{since } v_0 = 0$$

pumpkin 2

$$t_2 = \frac{v_0}{g} \mp \left[\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}\right]^{1/2}$$

Choose positive time ~~that is shortest~~

negative time is for up and down

Sufficient equations?

Work to target

$$\Delta t = t_1 - t_2$$

$$= \left(\frac{2h}{g}\right)^{1/2} - \left[\frac{v_0}{g} \mp \left[\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}\right]^{1/2}\right]$$

$$v_0 = -v$$

Check Units

$$[s] = \left(\frac{m}{m/s^2}\right)^{1/2} - \left[\frac{m/s}{m/s^2}\right] - \left(\frac{m^2/s^2}{m^2/s^4} + \frac{m}{m/s^2}\right)^{1/2}$$

$$= (s) - [s] - (s^2 + s^2)^{1/2}$$

$\boxed{\checkmark}$ units

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$\Delta t = \left(\frac{2h}{g}\right)^{1/2} + \frac{\cancel{v}}{g} \cancel{\mp} \left[\left(\frac{\cancel{v}}{g}\right)^2 + \frac{2h}{g}\right]^{1/2}$$

$$= \left(\frac{2h}{g}\right)^{1/2} + \frac{v}{g} - \left[\left(\frac{v}{g}\right)^2 + \frac{2h}{g}\right]^{1/2}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Is Answer Complete?

Why is Answer Reasonable?

- units check
- If $\cancel{v} = \emptyset$ then Δt should equal $\emptyset$ $\boxed{\checkmark}$
- If $v_0 \Rightarrow \infty$ then $\Delta t \to \left(\frac{2h}{g}\right)^{1/2}$ [drop time]
- If $2gh \ll v^2$ then use binomial approx $(1+x)^n \approx 1 + nx$ for $x \ll 1$

then

$$\Delta t \sim \left(\frac{2h}{g}\right)^{1/2} - \frac{h}{v}$$